CONSTITUTIVE MODEL FOR HIGH DAMPING RUBBER: EXPERIMENTAL FACTS AND MATHEMATICAL MODELING

S.I. Wiraguna[1], K. Yamada[2], A.F.M.S. Amin[3], Y. Okui[4]

ABSTRACT : The rate-dependent behavior of high damping rubber is investigated in shear and compression regime. A hyperelasticity model is proposed to represent the strain-rate independent elastic responses. A parameter identification scheme proposed earlier (Amin et. al. 2002) is adopted to identify the parameters of proposed hyperelasticity model for equilibrium and instantaneous responses. To do this, data gathered from compression and shear regime experiments are used simultaneously to obtain a unique set of material parameters capable of representing the behavior of rubber in compression and shear deformation as well. This hyperelasticity model is incorporated into a finite deformation viscoelasticity model to represent strain-rate dependent responses. The numerical results are compared with the experimental ones to show the adequacy of the proposed constitutive model and the parameter identification procedure.

KEYWORDS : High damping rubber, hyperelasticity, strain-rate independent, strain-rate dependent, viscoelasticity, constitutive model.

1. INTRODUCTION

In recent years, the use of rubber has increased substantially in engineering application. Now rubber forms one of the most important components of a wide range of products. It is used frequently in civil engineering structures because of its unique properties like high elasticity and large deformability in all deformation modes before undergoing a failure. In 1969, rubber was first used to devise a base isolation system for protecting an elementary school building in Skopje, Macedonia from earthquakes (Kelly 1997). Natural rubber (NR) was used for this base isolation device. Recently, high damping rubber (HDR) has been developed for better performance of such devices. This new type of rubber has a high proportion of filler, which is added during vulcanization process. As a result, upon cyclic loading, HDR has a better energy absorption than that of NR.

Under vertical loads, these base isolation devices are usually subjected to compression. However, under action of lateral loads like earthquake and wind, the device also experiences considerable shear deformation. In Kobe earthquake (1995) some of the structures having HDR bearings showed significantly better performance than the structures with other types of earthquake resistant devices. Due to this significant performance in protecting structures from real devastating earthquakes, the use of HDR bearings in earthquake resistant structures is getting popularity day by day. In order to design these bearings, two methods can be followed at present-full scale testing at the laboratory and numerical analysis on computers. When both the methods have their own difficulties, the later one is more suitable from design and performance evaluation point of view. Yet, the core of a successful numerical analysis lies on a reliable material model. Hence, in the industry, there is a long felt necessity regarding the formulation of an adequate constitutive model for HDR. Nevertheless, such a constitutive model for HDR should have the capability to perform well in compression and shear regimes.

Under monotonic loading, HDR displays typical rate dependent stress-strain responses. Figure 1 shows these responses schematically. The stress-strain curve will follow the equilibrium state (EOE' path) if HDR is loaded at an infinitely slow loading rate and will follow the instantaneous state (IOI' path) if it is loaded at an infinitely fast loading rate. The equilibrium and instantaneous states are elastic responses, and bound the viscosity domain. In the viscosity domain, the responses of HDR show the strain rate dependency effect. A viscoelasticity model incorporating an improved hyperelasticity model was proposed by Amin et al (2002) to simulate the responses of HDR in compression. The work is further extended here to improve the model capability in shear domain as well. To do this, simple shear experiments are carried out to investigate the mechanical behavior of HDR. The compression data is obtained from previous experiment (Amin et al 2002). This paper is focused on developing

[1]*M.Eng, Engineering Staff of PT. Yasa Patria Perkasa, Indonesia*
[2]*Graduate Student, Dept. of Civil and Environmental Eng. Saitama University, Japan*
[3]*Ph.D., Assistant Professor, Dept. of Civil Eng. Bangladesh University of Engineering and Technology, Bangladesh*
[4]*Dr. Eng., Associate Professor, Dept. of Civil and Environmental Eng. Saitama University, Japan*

the constitutive model of HDR to simulate the stress-strain relationship under simple shear and compression deformation.

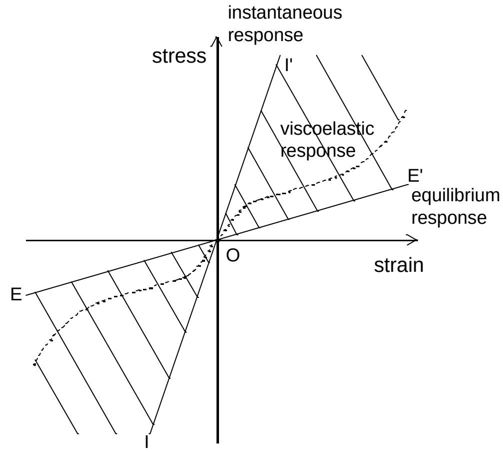
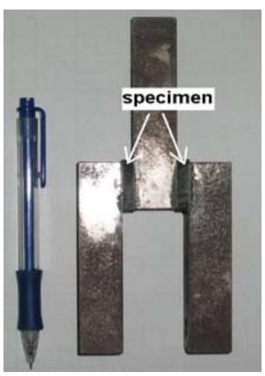


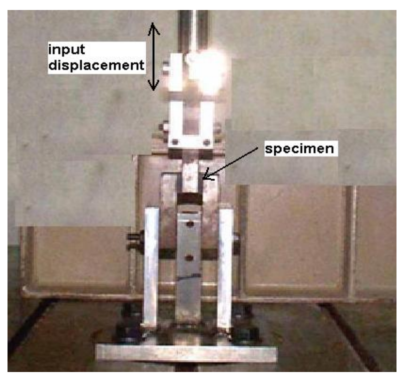
Figure 1. Typical stress-strain responses of HDR

2. SIMPLE SHEAR EXPERIMENT

Simple shear experiment is carried out to observe the rate-dependent behavior of HDR and to identify equilibrium response, instantaneous response, and viscoelastic response. To do this, the experimental scheme proposed earlier (Amin et al. 2002) in compression regime was followed. A computer-controlled servo-hydraulic testing machine (Shimadzu servo-pulser 4800) is used to test the specimens at room temperature. The input displacement is applied along the vertical axis, and the load cell measures the corresponding reaction as force. **Figure 2** shows the specimen and instrument of simple shear experiment. A preloading was applied on the virgin specimens before actual test to remove the Mullins' effect from other phenomena (Mullins 1969).



(a) Specimen



(b) Instrument

Figure 2. Specimen and instrument of simple shear experiment

To observe the equilibrium response, infinitely slow loading rate should be applied on a specimen. For HDR, it is very difficult to apply such a very slowly loading rate. Therefore, cyclic relaxation test is carried out to identify the equilibrium response by removing the time dependent effect. **Figure 3** shows the stress-strain response and strain history of cyclic relaxation test. From this test, equilibrium locus is obtained, which represents the equilibrium response of rubber.

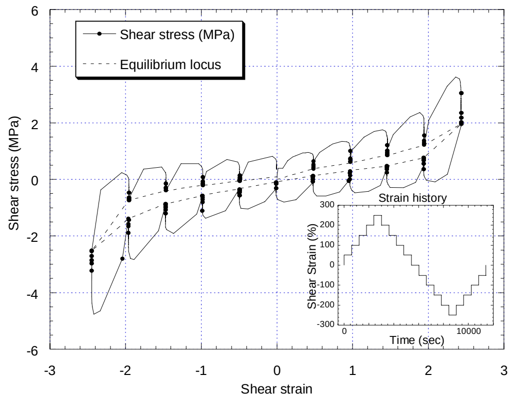


Figure 3. Strain history and stress-strain response in cyclic relaxation test

In order to trace the instantaneous response and to observe the strain-rate dependency of HDR, a series of cyclic tests with different strain rates are carried out. In theory, the instantaneous response is obtained by applying infinite first loading. However, in practice, it is impossible to apply the infinite first loading due to the limitation of a testing machine. In the actual tests, the strain rates from 0.05/s to 0.5/s are applied. **Figure 4** presents the stress-strain responses of monotonic loading path that was observed in four strain rate cases that show the strain-rate dependent responses. The equilibrium locus from cyclic relaxation test is also plotted to show the viscous domain. It is seen that the stress increases with increasing strain rates due to viscosity effect. However, at higher strain rates, the increase in the stress response due to the increase of strain rates gets deminished, and the stress responses tend to follow the same path. This indicates that the stress responses at higher strain rates reach the neighborhood of an instantaneous state.

In addition, simple relaxation test is also carried out to investigate viscosity of specimens, and to observe the relaxation phenomenon. **Figure 5** shows the stress history as well as the strain history of this test.

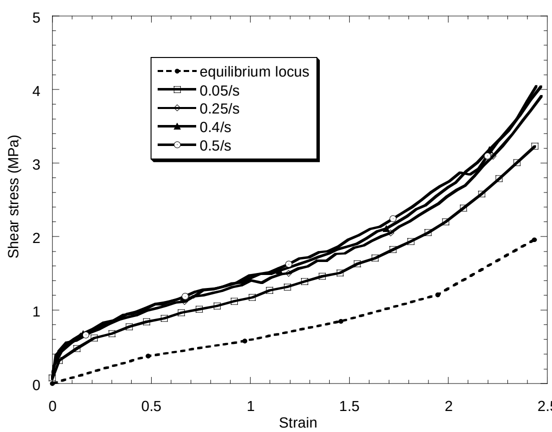


Figure 4. Monotonic path of shear at different strain rates

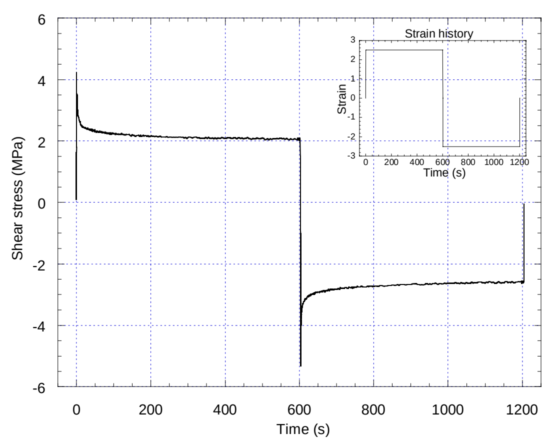


Figure 5. Strain history and stress history in simple relaxation test

3. CONSTITUTIVE MODEL

The elastic response of HDR can be modeled using a hyperelasticity model. In hyperelasticity, the stress-strain relationship is derived from a strain energy density function. For isotropic elastic materials, the strain energy density function W can be expressed as a function of invariants of a deformation invariant I_i, $(i=1,3)$, $W = W(I_1, I_2, I_3)$.

The deformation invariants can be written in terms of the left Cauchy-Green deformation tensor B, where, B = FF^T , and F is the deformation gradient tensor:

$$
\begin{aligned}
I_1 &= \mathrm{tr}B \\
I_2 &= \frac{1}{2}\{(\mathrm{tr}B)^2 - \mathrm{tr}(BB)\} \\
I_3 &= \det B
\end{aligned}
\qquad \textbf{(1)}
$$

Consideration of HDR as an incompressible material gives, $I_3 = 1$. Thus W is represented as a function of I_1 and I_2, $W = W(I_1, I_2)$.

From Truesdell and Noll (1992) it follows that the Cauchy stress (T) can be expressed as

$$
\begin{aligned}
\mathbf{T} &= -p\mathbf{1} + \mathbf{T}_{\mathbf{E}}, \\
\mathbf{T}_{\mathbf{E}} &= 2\frac{\partial W}{\partial I_1}\mathbf{B} - 2\frac{\partial W}{\partial I_1}\mathbf{B}^{-1},
\end{aligned}
\qquad \textbf{(2)}
$$

where, **1** is the identity tensor, p is the hydrostatic pressure and the subscript ‘E’ denotes the deviatoric part. From the above expression, T_{11E} and T_{21E} can be derived as

$$
\begin{aligned}
T_{11E} &= 2(\lambda_1{}^2 - \frac{1}{\lambda_1})(\frac{\partial W}{\partial I_1} + \frac{1}{\lambda_1}\frac{\partial W}{\partial I_2}) \\
T_{21E} &= 2\gamma(\frac{\partial W}{\partial I_1} + \frac{\partial W}{\partial I_2})
\end{aligned}
\qquad \textbf{(3)}
$$

where, λ is principal stretches, and γ is shear strain.

From these expressions **(1)-(3)** it is evident that the representation of incompressible hyperelastic behavior is solely dependent on the definition of W function. However, due to the strong dependence of W on the state of strain, experiments are required to identify the form of W. In this connection, Amin et al. (2002) carried out compression experiments to improve the Yamashita and Kawabata Model (1992) and thereby to identify the functional terms associated with I_1. In this paper, the authors report shear test results to include the other term associated with I_2 and having C_2 coefficient of the original Yamashita and Kawabat Model. The proposed strain energy density function W is presented in **Equation (4)**:

$$
W(I_1, I_2) = C_5(I_1 - 3) + C_2(I_2 - 3) + \frac{C_3}{N+1}(I_1 - 3)^{N+1} + \frac{C_4}{M+1}(I_1 - 3)^{M+1} \qquad \textbf{(4)}
$$

where, C_2, C_3, C_4, C_5, M, N are material parameters.

To model the viscosity effect of HDR, the proposed hyperelasticity model is incorporated into the finite deformation rate-dependent model. The three-parameter parallel model as illustrated in **Figure 6**, and finite deformation viscoelasticity law proposed by Huber and Tsakmakis (Huber and Tsakmakis 2000) is considered to develop the finite deformation viscoelasticity model. The hyperelastic element A represents the equilibrium response. The viscous dashpot B and hyperelastic element C represent the over-stress feature resulting from the rate-dependent effect. For simplicity, the conventional linear viscosity is assumed in this model.

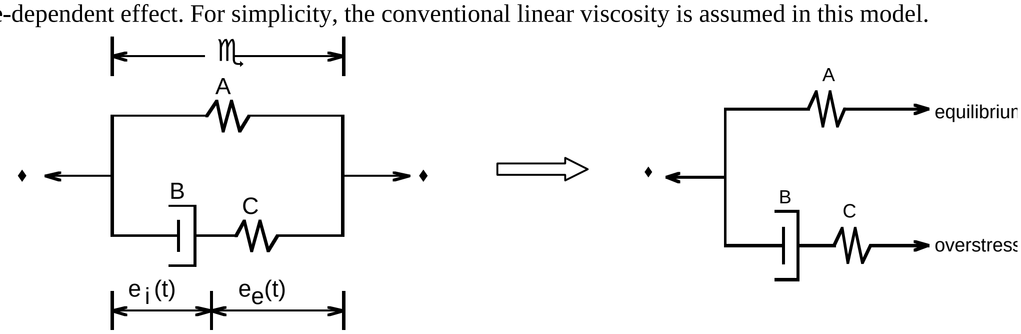


Figure 6. Three parameter parallel model

From three parameter parallel model in Figure 6, the Cauchy stress tensor can be decomposed into equilibrium part $T_E^{(E)}$ and the overstress part $T_E^{(OE)}$, $T_E = T_E^{(E)} + T_E^{(OE)}$. The total deformation gradient tensor F is decomposed into $F = F_e F_i$, where F_e and F_i are the deformation gradients associated with e_e and e_i, respectively. Then, the final form of Cauchy stress (T) and rate dependent part (B_e) are expressed as

$$\begin{aligned}
\mathbf{T} &= -p\mathbf{1} + 2\{C_5^{(E)} + C_3^{(E)}(\mathbf{I}_{1\mathbf{B}}-3)^{N^{(E)}} + C_4^{(E)}(\mathbf{I}_{1\mathbf{B}}-3)^{M^{(E)}}\}\mathbf{B} - 2C_2^{(E)}\mathbf{B}^{-1} + \\
&\quad 2\{C_5^{(OE)} + C_3^{(OE)}(\mathbf{I}_{1\mathbf{B}}-3)^{N^{(OE)}} + C_4^{(OE)}(\mathbf{I}_{1\mathbf{B}}-3)^{M^{(OE)}}\}\mathbf{B}_e - 2C_2^{(OE)}\mathbf{B}_e^{-1} \\
\dot{\mathbf{B}}_e &= \mathbf{B}_e\mathbf{L}^T + \mathbf{L}\mathbf{B}_e - \\
&\quad \frac{4}{\eta}\mathbf{B}_e\left(C_5^{(OE)}\mathbf{B}_e + C_3^{(OE)}(\mathbf{I}_{1\mathbf{B}}-3)^{N^{(OE)}}\mathbf{B}_e + C_4^{(OE)}(\mathbf{I}_{1\mathbf{B}}-3)^{M^{(OE)}}\mathbf{B}_e - C_2^{(OE)}\mathbf{B}_e^{-1}\right)^D
\end{aligned} \quad \textbf{(5)}$$

where, p is the hydrostatic pressure of T, **1** is the identity tensor, the subscript 'e' denotes the quantities related to F_e, L is the velocity gradient tensor, superscript 'D' denotes the deviatoric part. The subscripts B and Be denote the parts of the strain-invariants associated with e_e and e_i, respectively. η is the viscosity represented by the dashpot. Superscript 'E' denotes the equilibrium part, and the superscript 'OE' denotes the overstress part, as illustrated in **Figure 6**.

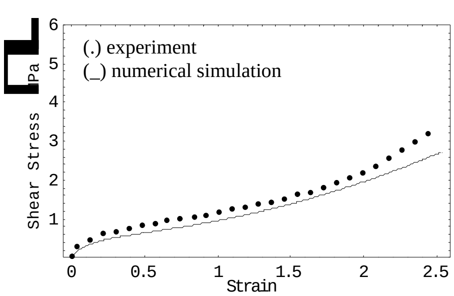
4. PARAMETER IDENTIFICATION AND NUMERICAL SIMULATION

The material parameters for equilibrium and overstress are determined from experimental results by using least square method according to **Equation (3)**. In this parameter identification procedure, the experimental results from compression and simple shear are simultaneously used to determine the coefficients associated with T_{11E} and T_{21E}, respectively. The parameters for overstress are obtained by subtracting the equilibrium parameters from the instantaneous parameters. **Table 1** presents the values of elasticity parameters. From **Equation (5)**, the remnant unknown parameter is only viscosity η. The simple relaxation results are used to estimate this viscosity parameter η according to the simulation trials of **Equation (5)**. Viscosity parameter η is found to be 4 MPa-s for compression and 25 MPa-s for shear.

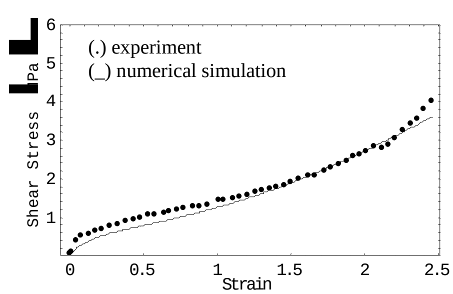
Table 1. Elastic material parameters

Responses	C_2 (MPa)	C_3 (MPa)	C_4 (MPa)	C_5 (MPa)	M	N
Equilibrium	0.145	1.182	-5.297	4.262	0.06	0.27
Instantaneous	0.166	2.477	-11.689	9.707	0.06	0.27
Overstress	0.021	1.296	-6.391	5.444	0.06	0.27

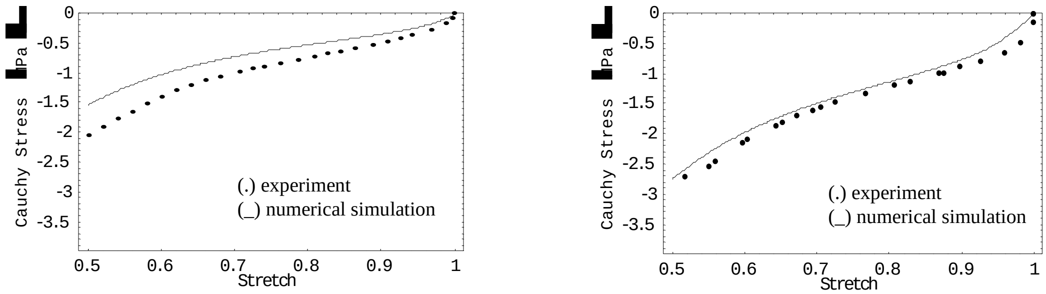
To verify the adequacy of the proposed constitutive model and the parameter identification procedure, the numerical simulations are compared with the experimental data. **Figure 7** shows the different strain-rate cases for shear and compression. From **Figure 7**, the capability of constitutive model to simulate the stress-strain response of HDR in shear and compression can be observed.



(a) Shear at 0.05/s



(b) Shear at 0.5/s



(c) Compression at 0.001/s **(d) Compression at 0.88/s**

Figure 7. Numerical simulation for different strain rates

5. CONCLUSIONS

A constitutive model is proposed to simulate the rate-dependent stress-strain response of HDR. According to the results of the simple shear and compressive experiments, the parameter identification scheme is proposed to identify of the material parameters of the constitutive model. Numerical simulation of test results show the adequacy of the proposed constitutive model and the parameter identification procedure to simulate the rate-dependent stress-strain response of HDR.

6. ACKNOWLEDGEMENTS

The authors are indeed grateful to Professor H. Horii, Department of Civil Engineering, University of Tokyo, Japan for extending experimental facilities of his laboratory to carry out the mechanical tests. Acknowledgements are also due to Yokohama Rubber Co., Japan for providing the specimens.

7. REFERENCES

Amin, A. F. M. S., Alam, M.S. and Okui, Y. (2002). "An improved hyperelasticity relation in modeling viscoelasticity response of natural and high damping rubbers in compression : experiments, parameter identification and numerical verification." *Mechanics of Materials*. 34, 75-95.

Huber, N. and Tsakmakis, C. (2000). "Finite deformation viscoelasticity laws." *Mechanics of Materials.* 32, 1-18.

Kelly, J. M. (1997). *Earthquake Resistant Design with Rubber*. Springer-Verlag, London.

Mullins, L. (1969). Softening of rubber by deformations, *Rubber Chem. Technol.,* 42, 339-362.

Truesdell, C., Noll, W. (1992). The non-linear field theories of mechanics. Second edition, Springer-Verlag, Berlin.

Yamashita, Y. and Kawabata, S. (1992). "Approximated form of the strain energy density function of carbon-black filled rubbers for industrial applications." *J. Soc. Rubber Ind. (Jpn).* 65, 517-528. *(in Japanese).*