CONSTITUTIVE MODEL OF HIGH DAMPING RUBBER

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ABSTRACT

Rubber materials have large deforming ability without damaging their internal structure. High damping rubber (HDR) is used in vibration isolator such as a rubber bearing for bridge etc. The mechanical behavior of HDR depends on strain rate, hysteresis, and temperture. The 3-parameter model generalized into the finite strain theory is used as constitutive model for HDR. It was attempted to observe hysteresis-dependent behavior of HDR experimentally to take this phenomena into a constitutive model. Finally, the hysteresis in rate-dependent behavior of HDR is not observed.
Keywords: rubber material, viscoelasticity model, stress relaxation, mechanical test

INTRODUCTION

General

In 1995, many civil engineering structures were damaged by Hyogoken-Nambu earthquake in Japan. For bridges, there was damage in many steel bearings. Therefore, rubber bearing that has large deforming ability came to use as an isolation device instead of a steel bearing. High damping rubber (HDR) is vulcanized natural rubber (NR) for isolation devices. Carbon black, silica and oil are added to HDR as special fillers during the vulcanization process to improve damping performance. Hence HDR has nonlinear rate-dependent behavior, hysteresis effect and damping performance. In order to rationalize design of HDR products, it is needed constitutive model for HDR.

Objective and methodology

Huber et, al [2] formulated finite strain theory for 3-parameter Maxwell model. Employing the theory, Amin et, al. [1] improved the hyper -elasticity relation for HDR on the basis of experimental observation and numerical simulation. Amin et, al. [3]

improved nonlinear viscosity in the model. Haupt et, al [4] observed hysteresis effect of filled rubber by experiments and modeled using elasto-plastic and Maxwell solids. In present paper, it is introduced the improved constitutive model for HDR introduced and shown experimental investigation about hysteresis effect of HDR are introduced.

CONSTITUTIVE MODEL

Hyperelasticity material

In the finite deformation theory, the strain energy rate per unit volume can be denoted by scalar products of conjugated stress and strain rates,

$$\dot{W} = \mathbf{S} : \dot{\mathbf{E}} = J\mathbf{T} : \mathbf{D}, \tag{1}$$

where W is the elastic potential function (W-function). $\mathbf{S}$ and $\mathbf{T}$ are Kirchhoff, Cauchy stress tensor, respectively. $\mathbf{E}$ and $\mathbf{D}$ are Green-Lagrange strain tensor and strain rate tensor. J is volume change and $J \equiv \det \mathbf{F} = 1$ under the condition of incompressibility. Amin et, al. [1] have proposed a W-function for HDR,

$$W(I, II) = C_5(I-3) + C_2(II-3) + \frac{C_3}{N+1}(I-3)^{N+1} + \frac{C_4}{M+1}(I-3)^{M+1}, \tag{2}$$

where C_5, C_2, C_3, C_4, M and N are the material constants. I and II are the first and second invariants of the left Cauchy-Green strain tensor $\mathbf{B} = \mathbf{F}\mathbf{F}^T$.

3-parameter model

The viscoelasticity model is used to represent the rate-dependent behavior of HDR. This section introduces a finite strain viscoelasticity model formulated by Huber et, al [2]. The 3-parameter Maxwell model (**Figure 1**) is generalized into the finite strain theory. It is based on the multiplicative decomposition of the deformation gradient. The decomposition of total strain ε into e_e and e_i can be denoted by the multiplicative decomposition of the deformation gradient $\mathbf{F}$ into an elastic part $\mathbf{F}_\mathrm{e}$ and an inelastic part $\mathbf{F}_\mathrm{i}$ in finite strain theory,

$$\mathbf{F} = \mathbf{F}_\mathrm{e}\mathbf{F}_\mathrm{i}. \tag{3}$$

The extra stress $\mathbf{S}_\mathrm{E}$ of the kirchhoff stress tensor $\mathbf{S}$ is the sum of the rate -independent equilibrium stress $\mathbf{S}_\mathrm{E}^{(\mathrm{E})}$ and a rate-dependent overstress $\mathbf{S}_\mathrm{E}^{(\mathrm{OE})}$ (**Figure 1**),

$$\mathbf{S}_\mathrm{E} = \mathbf{S}_\mathrm{E}^{(\mathrm{E})} + \mathbf{S}_\mathrm{E}^{(\mathrm{OE})}, \tag{4}$$

where the superscript “(E)” and “(OE)” are the quality related to equilibrium stress and overstress responses respectively. In this 3-parameter model shown in **Figure 1**, the springs can be models as the hyperelasticity. Then, we have

$$\mathbf{S}_{\mathrm{E}}^{(\mathrm{E})} = 2\frac{\partial W^{(E)}}{\partial I_B}\mathbf{B} - 2\frac{\partial W^{(E)}}{\partial II_B}\mathbf{B}^{-1}, \tag{5}$$

$$\mathbf{S}_{\mathrm{E}}^{(\mathrm{OE})} = 2\frac{\partial W^{(OE)}}{\partial I_{B_e}}\mathbf{B}_e - 2\frac{\partial W^{(OE)}}{\partial II_{B_e}}\mathbf{B}_e^{-1}, \tag{6}$$

$$\dot{\mathbf{B}}_e = \mathbf{B}_e\mathbf{L}^T + \mathbf{L}\mathbf{B}_e - \frac{2}{\eta}\mathbf{B}_e\left(\mathbf{T}_E - \mathbf{T}_E^{(E)}\right)^D, \tag{7}$$

where subscript "e" is the quality related to $\mathbf{F}_{\mathrm{e}}$. The relationship between $\mathbf{S}_{\mathrm{E}}$ and $\mathbf{B}$ can be obtained to substitute Eq. (2), (5), (6) into Eq. (4). The material constants in Eq. (2) are identified from experiments data defined from instantaneous and equilibrium responses (**Figure 2**) by using least-square method.

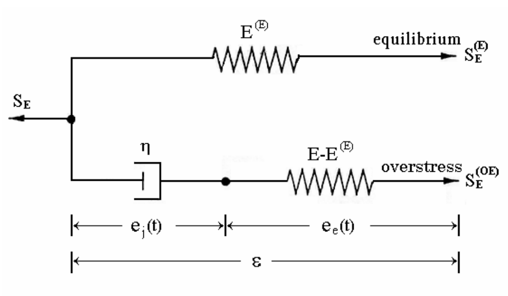

Figure 1. Decomposition of deformation and stress in 3-parameter model

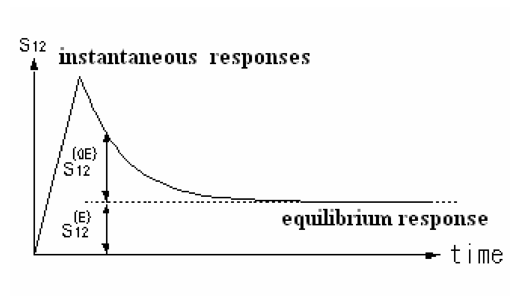

Figure 2. Instantaneous and equilibrium response under simple shear

Nonlinear viscosity

In the finite theory of 3-parameter solid, the relationship between the strain rate and the stress is expressed as

$$\left(\hat{\mathbf{P}}_E^{(OE)}\right)^D = \eta\hat{\mathbf{D}}_i, \tag{8}$$

where $\hat{\mathbf{P}}_E^{(OE)}$ is called Mandel stress tensor. Superscript "D" is the deviation component. "^" is the intermediate configuration. η is the viscosity of dashpot. Under the simple shear deformation, Mandel stress tensor is equal to the Kirhhoff stress tensor.

Amin et, al. [3] employed a nonlinear viscosity model to reproduce the rate-dependent behavior of NR and HDR. In that paper, the effects of the strain level and the strain rate on the viscosity are observed experimentally. They employ a power law to express the nonlinear viscosity η as a function the overstress $\hat{\mathbf{P}}_E^{(OE)}$ and left Cauchy-Green tensor $\mathbf{B}$:

$$\eta = \frac{\eta_0 \|\mathbf{B}\|^{\varphi}}{\left\|\hat{\mathbf{P}}_E^{(OE)}\right\|^{\delta}}, \tag{9}$$

where δ, φ and η_0 are material parameters to be defined and $\|\mathbf{\Omega}\| = \sqrt{\mathbf{\Omega} \cdot \mathbf{\Omega}}$ is the magnitude of tensor. At the same time, Eq. (10) is employed considering strong rate dependence during loading and weaker rate dependence during unloading respectively,

$$\eta = \left\{\eta_1 + (\eta_0 - \eta_1)\frac{1}{2}\{1 + \tanh\{\xi \mathbf{S} \cdot \mathbf{D}\}\}\right\} \frac{\|\mathbf{B}\|^{\varphi}}{\left\|\hat{\mathbf{P}}_E^{(OE)}\right\|^{\delta}}. \tag{10}$$

Finally, numeral simulations are performed (**Figure 5**) in Amin et, al. [3]

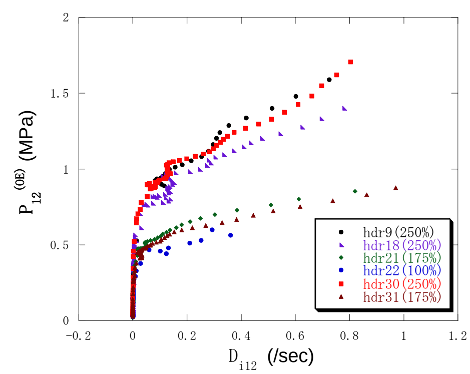

Figure 3. Strain level difference in relationship between overstresses and strain rate

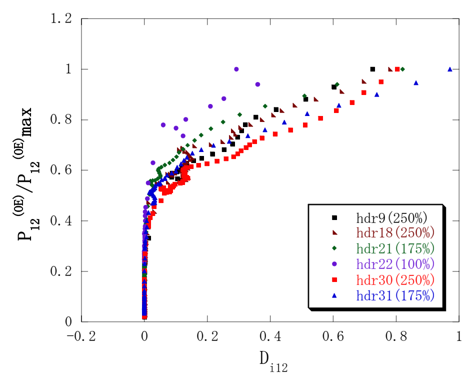

Figure 4. Relationship between normalized overstresses and strain rate

EXPERIMENT

Relaxation tests

This paper examines the relaxation behavior of the HDR under simple shear. To observe hysteresis in rate-dependent behavior of the HDR, deferent cyclic strain history is applied to specimens before relaxation tests. The HDR specimens were manufactured by the Yokohama Rubber Company, Japan. The tests were carried out using a computer-controlled servo hydraulic testing machine (Shimadzu Servo Pulser). The simple shear specimens (25 mm · 25 mm · 5 mm) have a net shear area of 25 mm^2 (**Figure 6**). All tests were carried out at room temperature. Before an actual test, each virgin specimen was subjected to a five-cycle preloading process to remove the Mullins softening effect. In preloading for shear tests, a strain rate of 0.05/s was applied for each cycle with maximum shear strain of 2.5. All tests were conducted 20 minutes after completing the preloading to regularize the healing effect. Each test was

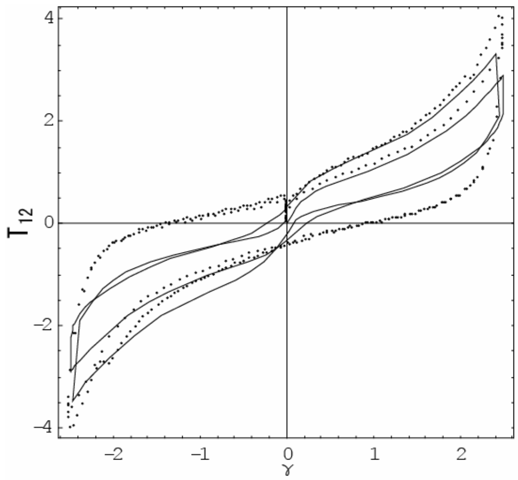

Figure 5. Expriments and simulation results by Amin under shear

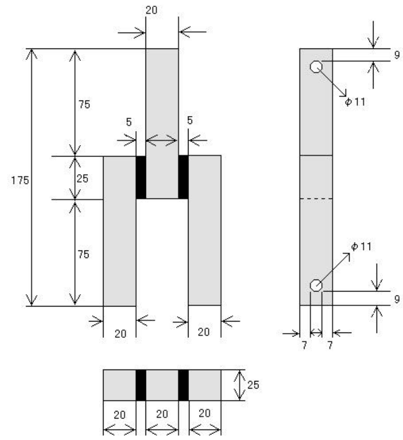

Figure 6. Shear specimen of HDR

conducted with a new specimen that only contained the history of the preloading procedure.

After healing time, the relaxation behavior at different strain history is examined by a simple relaxation (**Figure 7**). During the cycle loading to apply strain history, strain rate is 0.05/s. During the simple relaxation tests, stress relaxation was recorded for 3600 s after giving the 250% strain with the 3.6/s strain rate. The number of cyclic loading before a relaxation test is assigned to 0, 5, 10, and 20. In **Figure 7**, softening behavior of HDR can be seen with strain history.

Experimental results

Haupt et, al [4] observed the change of relaxation rate and equilibrium stress of the filled rubber caused by hysteresis effect. The phenomenon was expressed by nonlinear viscosity including internal variables in their model. **Figure 8** shows the relationship between the over stress $\hat{\mathbf{P}}_{E12}^{(OE)}$ and the strain rate $\hat{\mathbf{D}}_{12}$ for the relaxation tests with the different numbers of loading cycles. The rate-dependent behavior of HDR differs depending on the stress history. In **Figure 9**, the overstress is normalized by its maximum value.

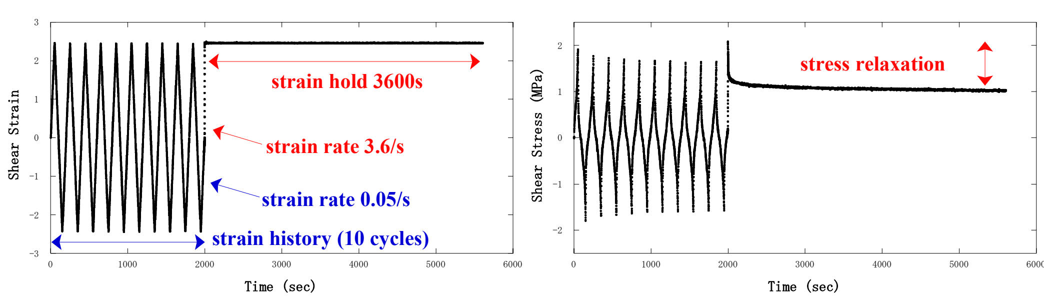

Figure 7. Strain input (left) and stress out put (right) in 10 cycles strain history

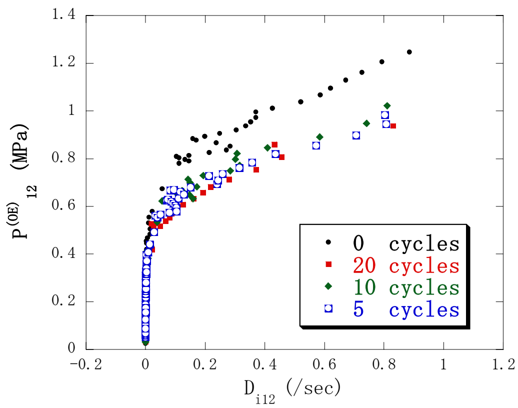

Figure 8. Strain history difference in relationship between overstresses and strain rate

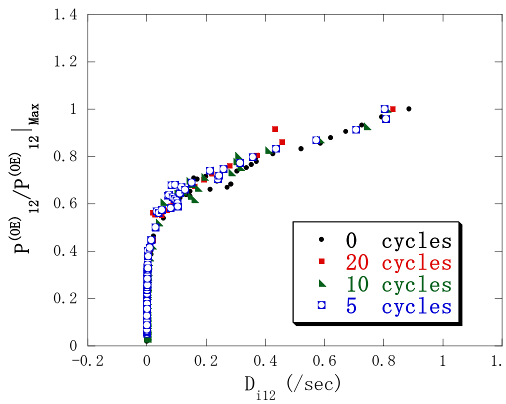

Figure 9. Relationship between normalized overstresses and strain rate

SUMMARY

In this paper, a constitutive model for high damping rubber using finite strain theory of 3-parameter Maxwell model is introduced. By performing relaxation tests with different strain history, the hysteresis in rate-dependent behavior of HDR is investigated. As far as the present experimental results, the hysteresis in rate-dependent behavior of HDR is not observed.

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