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Nonlinear viscosity law for rate-dependent response of high damping rubber: FE implementation and verification

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ABSTRACT: A simple computational strategy for implementation of a finite strain viscohyperelasticity model in a standard FEM code is discussed. To this end, a recently published evolution law that takes the nonlinear dependence of viscosity into account is considered. Such a law considers the maximum overstress experienced in past history and deformation as the internal variables. In order to simulate the stress-response for a particular boundary value problem, an analytical solution scheme using the Bernoulli's principle has been applied to derive the stress expressions. The expressions have been incorporated in a finite element code. Limited numerical trials show the applicability of the procedure for simulating rate dependent responses obtained from one dimensional experiments e.g. uniaxial compression or simple shear tests on natural rubber and high damping rubber specimens. The limitations of this approach for solving an arbitrary boundary value problem are also discussed.

1 INTRODUCTION

Base isolation technique is a well-known method for protecting structures from earthquake induced damages. Use of vulcanized rubber in constructing base isolation bearings are being increasingly practiced during the last decades. To this end, the rubber industries in Japan have pioneered the development of specially vulcanized rubbers with high damping properties, commonly known as high damping rubber (HDR). Nevertheless, such a vulcanization procedure not only induces the damping property but also significantly enhances other mechanical properties e.g. nonlinearity in stress responses, strain-rate dependency, hysteresis etc. of the material itself. The HDR bearing with reinforced steel plates supports the structure by restricting the bulging feature of rubber layers and reduces the inertia force of the structure through the dynamics of the system (Skinner et al. 1993). Since the development of HDR materials and HDR bearings, many researchers were motivated to study the mechanical behavior of the material and the bearing system as a whole including the FE simulation of the full scale bearings as well. The very initial works described in Seki et al. (1987), Takayama et al. (1990), Billings (1993), Kelly (1997), Matsuda (1999), Yoshida et al. (2004) & Ali and Abdel Gaffar (1995) can be noted. Yet all these studies had severe limitations towards appreciating the rate-dependency effect that significantly exists in HDR. To this end, the research group concentrating around Saitama University, Japan directed their extensive efforts to study rate-dependency effect, experimental characterization, and development of evolution equations for nonlinear elasticity and viscosity effects and identification of constitutive parameters through experiments (Amin 2001, Amin et al. 2002, Wiraguna (2003) & Amin et al. 2006a, b. Through these successive efforts it was possible to propose an improved hyperelasticity relation for representing the rate-independent responses and also to propose a nonlinear viscosity law based on internal variables to represent the rate-dependent responses in compression and shear regimes. Bhuiyan (2004) implemented the improved hyperelasticity model as proposed in Wiraguna (2003) in a finite element code (see also Amin et al. 2006a). There has been shown that all these models can produce simulation results with good conformity with the responses obtained from

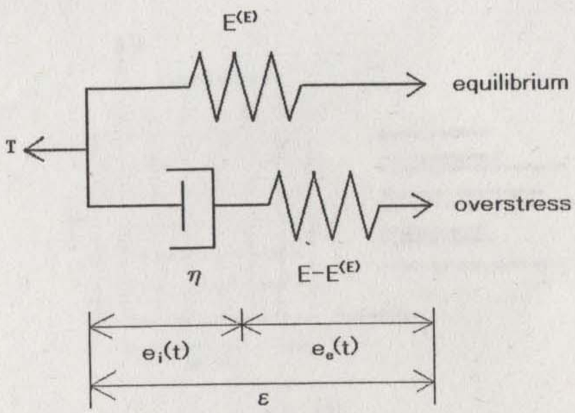


Figure 1. Three-parameter parallel model.

mechanical tests. In this course, the present work is motivated towards the finite element implementation of the nonlinear viscosity relation in a finite element code as an extension to the earlier contributions of this research group. This presentation discusses the approaches that we followed implementing the nonlinear evolution equation for viscosity in a finite element code for simulating the one dimensional experiments.

2 CONSTITUTIVE MODELING

A 3- parameter Maxwell model as shown in Figure 1 is used to model HDR. In Figure 1 the total stress is decomposed into two parts i.e. rate independent equilibrium part and rate dependent overstress part. To model the rate dependency phenomenon of HDR, the hyperelastic models are required to combine with rate dependent model. In this work, the improved hyperelasticity model proposed by Amin et al. (2002) & Wiraguna (2003) has been used to combine with rate-dependent model (Huber and Tsakmakis 2000) in order to determine the total stress-strain relation of HDR.

In this context, Equation (1) representing the strain energy density function, W, expressed as a function of the invariants of deformation tensor of HDR material considering as an incompressible and isotropic-elastic material, can be used in the current work.

$$W(I_1, I_2) = C_5(I_1 - 3) + \frac{C_3}{N+1}(I_1 - 3)^{N+1} + \frac{C_4}{M+1}(I_1 - 3)^{M+1} + C_2(I_2 - 3) \tag{1}$$

where, C_2, C_3, C_4, C_5, M and N are material parameters. The invariants of the deformation tensor can be written in terms of the principal stretches $\lambda_i (i = 1, 2, 3)$,

$$I_1 = \mathrm{tr}\,\mathbf{B} = \lambda_1^2 + \lambda_2^2 + \lambda_3^2$$

$$I_2 = \frac{1}{2}\left\{(\mathrm{tr}\mathbf{B})^2 - \mathrm{tr}(\mathbf{BB})\right\} = (\lambda_1\lambda_2)^2 + (\lambda_2\lambda_3)^2 + (\lambda_3\lambda_1)^2$$

From Truesdell and Noll (1992), the Cauchy stress **T** can be expressed as

$$\mathbf{T} = -p\mathbf{1} + \mathbf{T}_E \tag{2}$$

$$\mathbf{T}_E = 2\frac{\partial W}{\partial I_1}\mathbf{B} - 2\frac{\partial W}{\partial I_2}\mathbf{B}^{-1} \tag{3}$$

where **1** is the identity tensor, p is the hydrostatic pressure that can be determined from the boundary condition and the subscript 'E' defines the deviatoric part of the stress.

From the model structures shown in Figure 1, the deviatoric part of the Cauchy stress tensor can be written as the sum of the equilibrium part $\mathbf{T}_E^{(E)}$ and the overstress part $\mathbf{T}_E^{(OE)}$:

$$\mathbf{T}_E = \mathbf{T}_E^{(E)} + \mathbf{T}_E^{(OE)} \tag{4}$$

with

$$\mathbf{T}_E^{(E)} = 2\frac{\partial W^{(E)}}{\partial I_{1\mathbf{B}}}\mathbf{B} - \frac{\partial W^{(E)}}{\partial I_{2\mathbf{B}}}\mathbf{B}^{-1} \tag{5}$$

$$\mathbf{T}_E^{(OE)} = 2\frac{\partial W^{(OE)}}{\partial I_{1\mathbf{B}_e}}\mathbf{B}_e - \frac{\partial W^{(OE)}}{\partial I_{2\mathbf{B}_e}}\mathbf{B}_e^{-1} \tag{6}$$

where $\mathbf{B} = \mathbf{F}\mathbf{F}^T$, $\mathbf{B}_e = \mathbf{F}_e\mathbf{F}_e^T$ and $I_{1\mathbf{B}}$ and $I_{2\mathbf{B}}$ are the first and second invariants of the **B**. the subscript 'e' denotes the quantities related to $\mathbf{F}_e$.

Following the concept of Huber and Tsakmakis (2000), the rate of left Cauchy-Green deformation tensor can be expressed as

$$\dot{\mathbf{B}}_e = \mathbf{B}_e\mathbf{L}^T + \mathbf{L}\mathbf{B}_e - \frac{2}{\eta}\mathbf{B}_e\left(\hat{\mathbf{P}}_E - \hat{\mathbf{P}}_E^{(E)}\right) \tag{7}$$

The (·) indicates the material time derivative, η is the material viscosity, $\mathbf{P}_E$ is Mandel stress tensor and **L** is the velocity gradient expressed as,

$$\mathbf{L} = \dot{\mathbf{F}}\mathbf{F}^{-1} \tag{8}$$

3 EVOLUTION OF NONLINEAR VISCOSITY

Recently, Amin et al. (2006b), following the general constitutive theory based on Huber and Tsakmakis (2000), proposed an explicit description of the evolution equation of nonlinear viscosity by analyzing the experimental data of compression and shear. Equation (9) represents the constitutive equation of viscosity of the power law type (Amin et al. 2006a) in general three dimensional form

$$\hat{\mathbf{D}}_i = \frac{\left\|\hat{\mathbf{P}}_E^{(OE)}\right\|^\delta}{\eta_0 \pi^\delta \|\mathbf{B}\|^\varphi} \cdot \hat{\mathbf{P}}_E^{(OE)} \tag{9}$$

where φ, δ, η_0 are material parameters to be determined and $\|\omega\| = \sqrt{\omega \cdot \omega}$ is the magnitude of a tensor. The closed form of the evolution equation of the constitutive equation of nonlinear viscosity can be obtained from Equation (9).

$$\frac{1}{\eta\left(\hat{\mathbf{P}}_E^{(OE)}, \mathbf{B}\right)} = \frac{1}{\eta_0}\left(\frac{\left\|\hat{\mathbf{P}}_E^{(OE)}\right\|}{\pi}\right)^\delta \|\mathbf{B}\|^{-\varphi} \tag{10}$$

In equation (9) and (10) the parameter $\pi = (1\text{ MPa})$ has been introduced for dimensional reasons (Amin et al. 2006b).

4 COMPUTATIONAL STRATEGY

The evolution law [Equation (7)] is a first order differential equation on $\mathbf{B}_e$. The input variable is the deformation gradient tensor $\mathbf{F}$. The left Cauchy–Green deformation tensor is straightforward $\mathbf{B} = \mathbf{F}\mathbf{F}^T$. Let $\mathbf{B}_e$ be the internal state variable.

The equation (7) can be rewritten in indicial form as

$$\dot{B}_{eij} = B_{eij} L_{ij}^{\ T} + L_{ij} B_{eij} - \frac{4}{\eta} B_{eij}^2 C \tag{11}$$

where

$$C = C_5^{(OE)} + C_3^{(OE)}\left(I_{\mathbf{B}_e} - 3\right)^N + C_4^{(OE)}\left(I_{\mathbf{B}_e} - 3\right)^M$$

and $(i, j) \in \{1,2,3\}$

and the material parameters such as C_5, C_3, C_4, M and N as determined by Amin (2001) are used in this work for simulation purpose. Expanding the evolution equation leads to a set of 6 differential equations with 6 unknowns: (B_{e11}, B_{e22}, B_{e33}, B_{e12}, B_{e23}, B_{e13}). Now the Equation (11) can be formulated as the standard ordinary differential equation

$$\frac{dy}{dx} + p(t).y = Q(t).y^2 \tag{12}$$

where $P(t) = -\left(L_{ij}^{\ T} + L_{ij}\right)$, $Q(t) = -\frac{4}{\eta} C$ and $B_{eij} = y$

Table 1. Elasticity and viscosity parameters of HDR.

	Response	C_2 (MPA)	C_3 (MPA)	C_4 (MPA)	C_5 (MPA)
HDR	Equilibrium	0.145	1.182	−5.297	4.262
	Overstress	0.021	1.295	−6.392	5.445

	M	N	η_0 MPa–s	φ	δ
HDR	0.06	0.27	1.63	2.29	1.46

The above equation is of the typical form of Bernoulli's equation which is given as,

$$\frac{dy}{dt} + P(t) \cdot y = Q(t) \cdot y^n \tag{13}$$

By means of the substitution $z = y^{-n+1}$, Equation (13) may be written as,

$$\frac{dz}{dt} + (1-n) \cdot P(t) \cdot z = (1-n) \cdot Q(t) \tag{14}$$

This is linear equation of the first order.

Now using Equation (15) and recalling $y = B_{eij}$, the solution of Equation (7) becomes

$$B_{eij} = \frac{1}{\frac{1}{B_{ij}} * e^{-\left(L_{ij}^{\ T} + L_{ij}\right)t} + \frac{4C}{\eta} \cdot t} \tag{15}$$

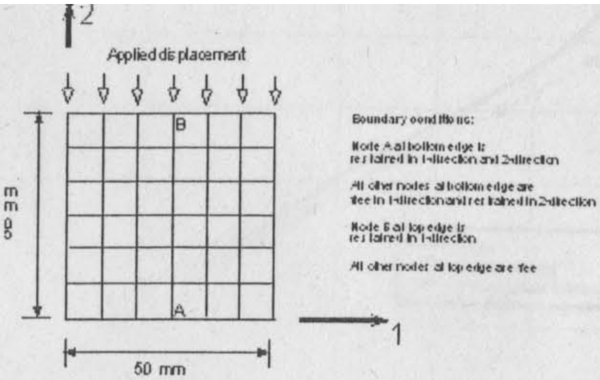
Equation (15) will be used to determine the Cauchy stress and thus implemented in the general purpose finite element code to simulate the experimental results along with those obtained using the constitutive relation.

5 MATERIAL PARAMETERS

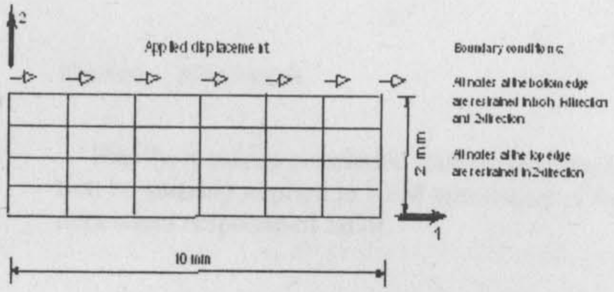
Equation (1) includes material parameters (C_2, C_3, C_4, C_5, M and N) to represent the strain energy density function and Equation (10) includes $\boldsymbol{\varphi}$, $\boldsymbol{\delta}$, η_0 to states the evolutionary equation of nonlinear viscosity function. Table 1 shows the description of these material parameters as determined by Amin et al. (2002) & Wiraguna (2003) for the purpose of the simulation.

6 FINITE ELEMENT SIMULATION

The analytical solution strategy of evolution equation [Equation (7)] as described in Section 5 is used



(a)



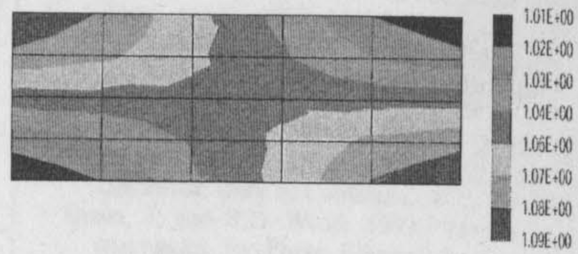
(b)

Figure 2. FEM model with geometric boundary (a) 216-brick-element for compression test (b) 100-brick-element for shear.

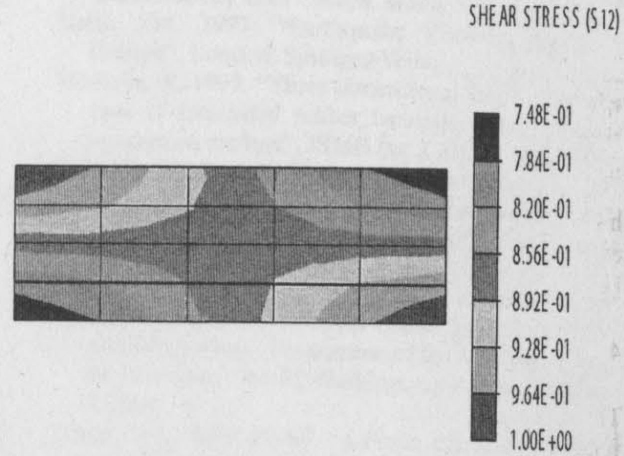
to formulate the FE code for incorporation in a versatile finite element program FEAP (Taylor 2006). A 3D finite element analysis is carried out using the FE models as shown in Figure 2(a) and (b). Eight-node brick element as available in FEAP is used to model the rubber. Both the geometric and material nonlinearities of rubber layers are considered in the analysis.

7 DISCUSSIONS

The main approach of the current work was to solve the viscosity induced evolution equation [Equation (7)] and thereby to implement the same in a general purpose finite element code to simulate the rate dependent responses (Amin et al 2006b). The experimental results and constitutive parameters as identified and reported in Amin et al. (2006a, b) were utilized here for simulation and comparison purposes. Figure 3(a) and (b) show the stress patterns obtained for the combined action of compression and shear using the rate independent-model and rate dependent model, respectively. A logical stress patterns have been observed in both cases. Figure 4(a) and (b) show the FE simulation results of simple relaxation test in shear and compression, respectively, where a good conformity

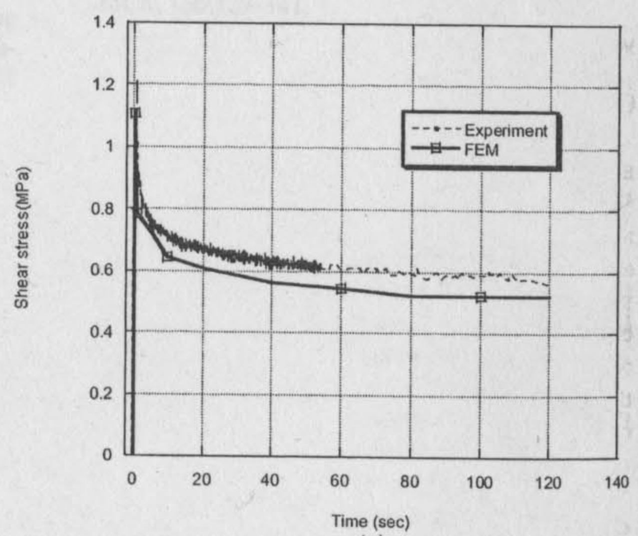


(a)



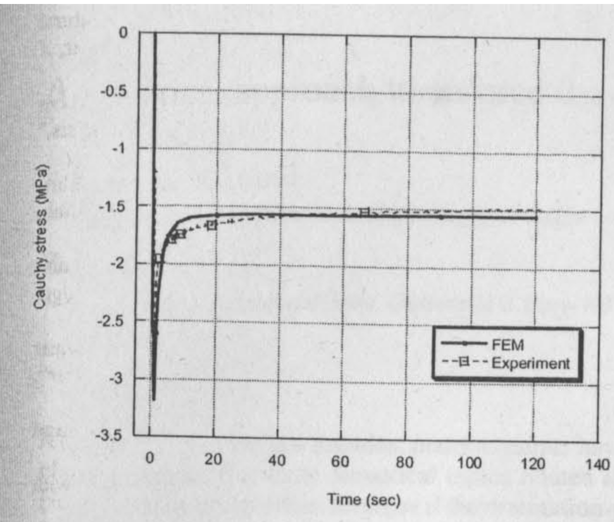
(b)

Figure 3. Stress pattern obtained from FEM simulation using (a) Rate-independent model (b) Rate-dependent model.



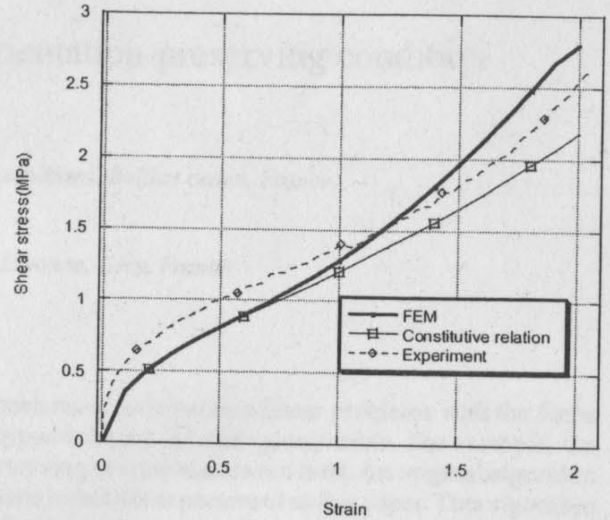
(a)

Figure 4. Simulation result of simple relaxation test (a) simple shear at strain level 1.0 (b) uniform compression at stretch level 0.5.



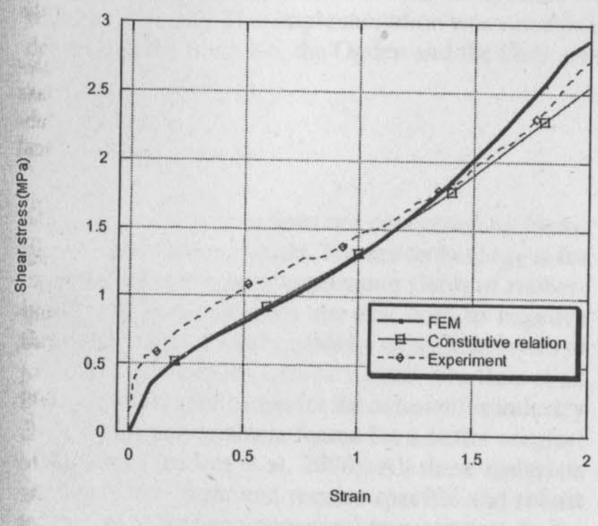
(b)

Figure 4. (Continued).



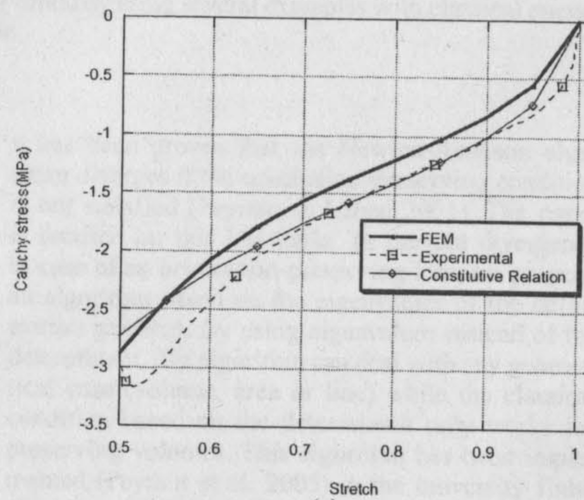
(b)

Figure 5. (Continued).



(a)

Figure 5. FEM simulation results of simple shear (a) 0.5/s (b) 0.25/s.



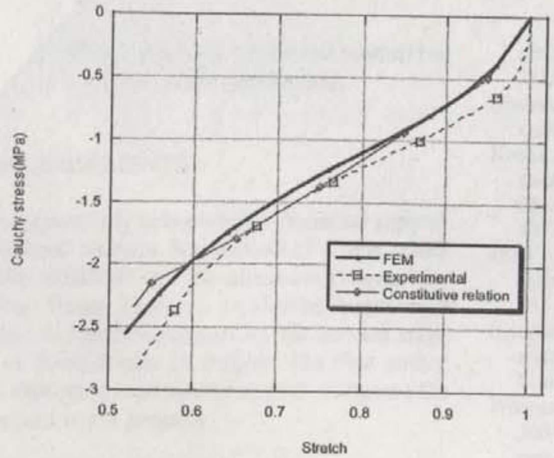
(a)

Figure 6. FEM simulation of compression (a) 0.88/s (b) 0.24/s.

with experiments and constitutive relations can be observed. Figure 5 shows the simulation results of simple shear at different stretch rates while Figure 5(a) and (b) show the simulation of the shear stress-strain response at 0.5/s and 0.25/s strain rates, respectively. Figure 6 shows the same for uniaxial compression cases. Although at the higher strain rate the simulation results as shown in Figure 5(a) have a good conformity with those of the experiment and constitutive relation ship but at lower strain rate it does not so as shown in Figure 5(b). Figure 6(a) and (b) show the compressive stress-strain response at 0.88/s and 0.24/s strain rates, respectively. From these figures, it is found to have a reasonably good conformity between experiment and FE simulation. However, at the lower strain rate a better agreement is well observed.

8 CONCLUSION

In the current work, the performance of the proposed mathematical solution has been justified for one dimensional experimental result only. The performance study of the current solution for general loading case needs to have more elaborated study as the next step of the current research work.



(b)

Figure 6. (Continued).

Finally, it can be concluded that the proposed solution be suitably applied in FEM simulation of the rate dependent response of HDR.

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