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AN ECONOMIC DESIGN APPROACH FOR HELICOIDAL STAIR SLABS BASED ON FINITE ELEMENT ANALYSIS

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Abstract

Eight-node curved thick shell finite elements were used to analyse two prototype helicoidal stair slabs without any geometric idealisation. The results of finite element analysis were compared with those obtained from traditional helical girder solution. The comparative investigation revealed that helical girder solution largely over estimates the vertical moment, lateral moment, lateral shear force and thrust along with an under estimation of torsion. Finally, for critical economic assessment, one of the prototype stairs was designed following ACI ultimate strength design method. The design exercise indicated that around 47% saving of the reinforcement required in resisting the moments and thrust can be achieved in FE approach with an overall economy of around 17%.

1 Introduction

The helicoidal stair has an inherent fascinating appearance among different forms of stairs from architectural point of view. For this reason, helicoidal stair slabs are increasingly being used nowadays in many important buildings. Geometrically, a helicoidal surface is a three dimensional structure in space consisting of a warped surface which is generated by moving a straight line touching a helix so that the moving line is always perpendicular to the axis of the helix. In an oblique helicoid, the generating line always maintains a fixed angle with the helix. Because of this complex geometric configuration, the traditional methods of analysis of helicoidal stairs are based on various idealisations and assumptions. There are two basic approaches.

In the first approach [1], the simplest solution is produced by reducing the helicoid to its horizontal projection and resolving the problem into that of a fixed ended curved beam. The structure is thus idealised as a two dimensional structure.

The second approach [2, 3, 4, 5] considers the helicoid as a helical girder (a space structure). In this approach, the helicoid is reduced to its elastic line having the same stiffness as that of original structure. Comparative assessment [6] of these two approaches showed that curved beam solution [1] leads to a very conservative estimation of forces.

The efforts on the development of an 'exact' procedure of analysis of helicoidal stair reached its culmination through the works of Santathadaporn and Cusens [7], where the stair was assumed as a helical girder. The work presented thirty six design charts for helical stairs of a wide range of geometric parameters. Based on this work, four design charts were compiled in somewhat modified form in current design hand books [8]. These design charts now stand as 'helical girder solution' for helicoidal stairs.

But both curved beam and helical girder solution fail to take into account the three dimensional characteristics of helicoid and its inherent structural efficiency. With a view to developing an 'exact' and general solution, Menn [9] outlined an analytical method of solving helicoidal shell problems including edge perturbations or edge conditions. It was observed that the analysis of a helicoidal shell for certain boundary conditions is possible through highly complex mathematical calculations. Menn realised the fact and concluded finally to go for 'girder solution'.

The situation has now changed. The development of different general curved shell elements in the field of finite element (FE) techniques and the availability of high speed digital computers at design engineers' desk have ushered in a new hope for the shell solution of this problem by FE method in a more logical and convenient way. But until now, no effort in this direction has been reported regarding the behaviour of this structure under uniformly distributed vertical loadings.

The main objective of the present investigation was therefore aimed to study the actual behaviour of the helicoidal stair slab in FE approach without any geometric idealisation and to make a comparative assessment between the helical girder solution [8] and the FE method of analysis in terms of economy, as earlier the curved thick shell element [10, 11, 12, 13] performed well in developing an economic design basis for dog-legged stairs [14, 15] and free standing stairs [16]. Based on this experience, the thick shell element was used to analyse two prototype fixed ended reinforced concrete helicoidal stair slabs. The stairs were analysed for the given geometry as helical girders as well. Finally, for an economic assessment, one of the prototype stairs was designed by using the results of both approaches. The paper presents some significant results of the work.

2 Prototype Stairs

The helicoidal stairs at Kamalapur Railway Station (Stair-I) and Sena Kalyan Bhaban (Stair-II) in the city of Dhaka, Bangladesh were selected for the present study. These two stairs are in service. The geometric dimensions of these structures were surveyed and are tabulated in Table 1 along with specified loading condition.

Parameters	Stair-I	Stair-II
Inner radius, m	1.52	0.69
Outer radius, m	3.80	2.41
Height of the stair*, m	3.73	3.40
Waist thickness, m	0.25	0.25
Central angle (on plan), degree	180.00	280.00
Height of the risers, m	0.15	0.15
Average step width, m	0.33	0.36
Clear cover, m	0.03	0.03
Uniformly distributed live load, N/m^2	4790.00	4790.00

* *Difference of elevation between centre of bottom support and centre of top support*

Table 1: Geometry of the prototype stairs.

3 Stress Resultants

In general, six stress resultants, i.e. vertical moment, lateral moment, torsion, thrust, lateral shear force and radial horizontal shear force are found on any cross section of a helicoid, as shown in Figure 1.

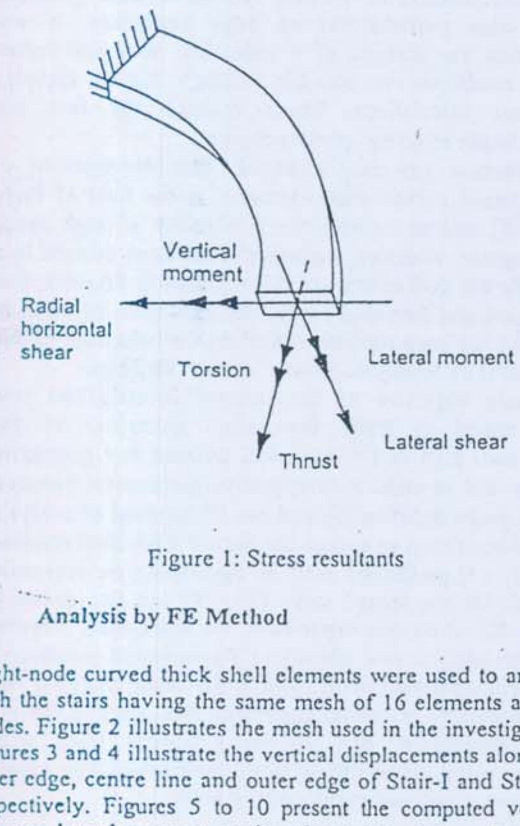

Figure 1: Stress resultants

4 Analysis by FE Method

Eight-node curved thick shell elements were used to analyse both the stairs having the same mesh of 16 elements and 69 nodes. Figure 2 illustrates the mesh used in the investigation. Figures 3 and 4 illustrate the vertical displacements along the inner edge, centre line and outer edge of Stair-I and Stair-II, respectively. Figures 5 to 10 present the computed vertical moment, lateral moment, torsion, thrust, lateral shear force and radial horizontal shear force along the span of Stair-I and Figures 11 to 16 present those of Stair-II due to the application of dead and live loads. The helical girder solutions have simply been superimposed here for comparison.

5 Findings of the FE Investigation

5.1 Deflection Characteristics

For helicoids with central angle of 180^0, the maximum deflection (Figure 3) occurred at the mid span. But in case of 280^0 stair, the maximum deflection (Figure 4) occurred at points away from the mid span with a pseudo fixity at the mid span. This interesting feature may be visualised as the cantilever action of two halves of the stair. The appearance of this fixity depends mostly on the central angle and waist thickness of the stair slab. In both cases, the slabs showed a tendency of tilting in the outward direction, which is quite logical for such an eccentric structure. The deflection characteristics of helicoidal stair slab obtained from FE analysis have been found to conform with the findings of two model studies [18, 19].

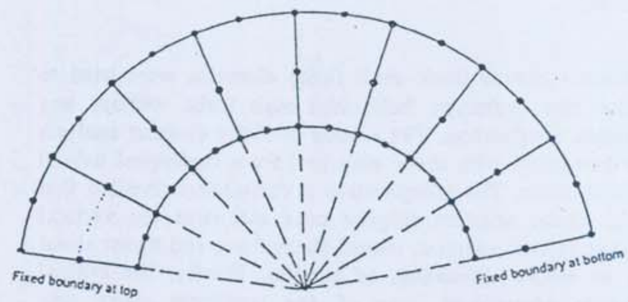

Figure 2: FE mesh used in the investigation

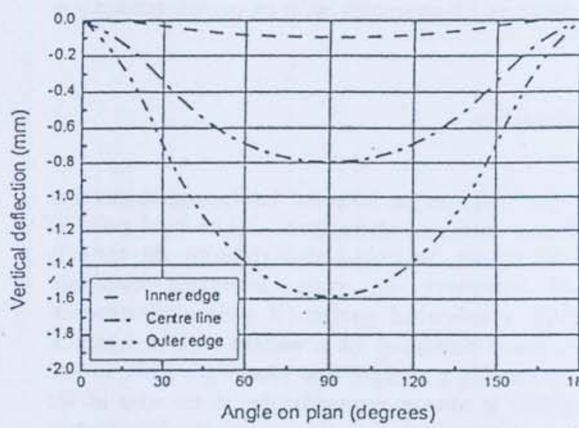

Figure 3: Vertical displacements (Stair-I)

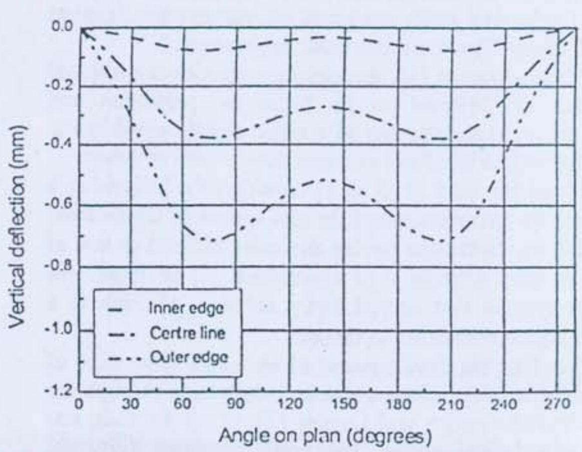

Figure 4: Vertical displacements (Stair-II)

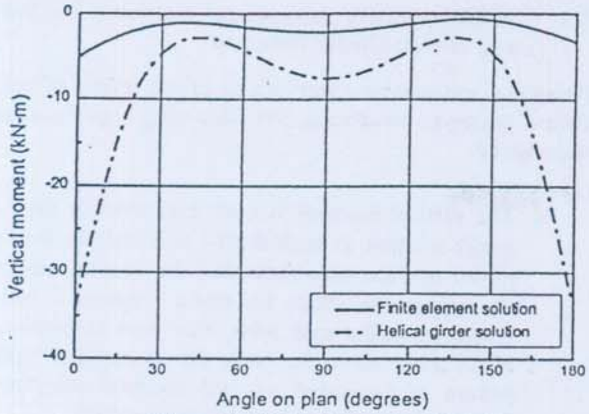

Figure 5: Vertical moment (Stair-I)

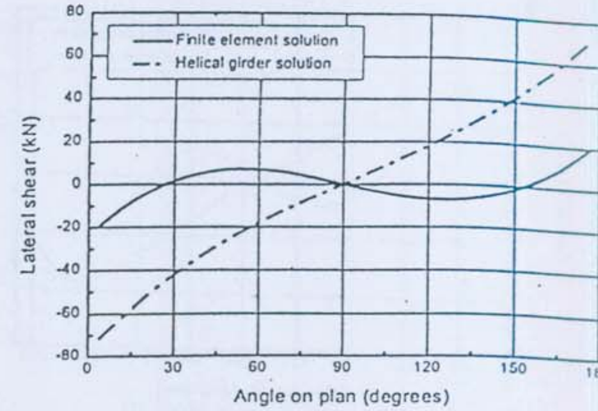

Figure 9: Lateral shear (Stair-I)

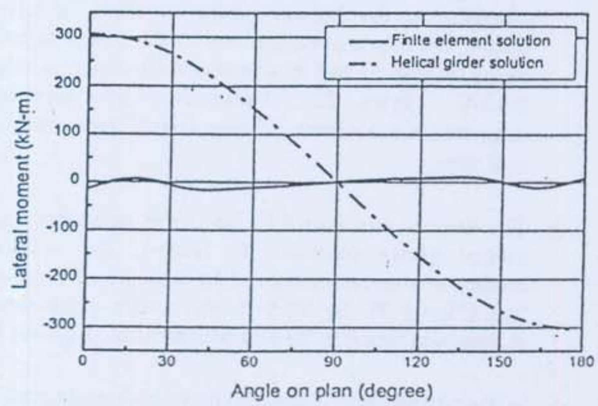

Figure 6: Lateral moment (Stair-I)

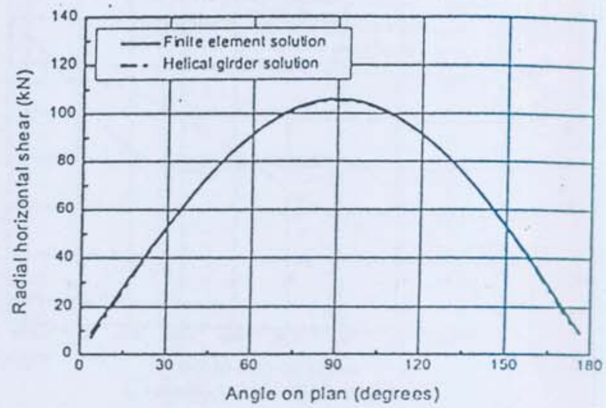

Figure 10: Radial horizontal shear (Stair-I)

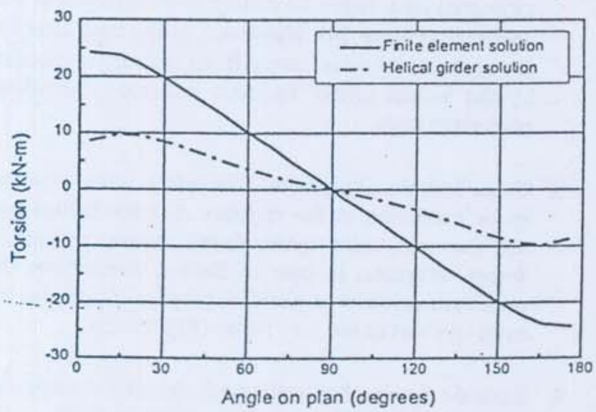

Figure 7: Torsion (Stair-I)

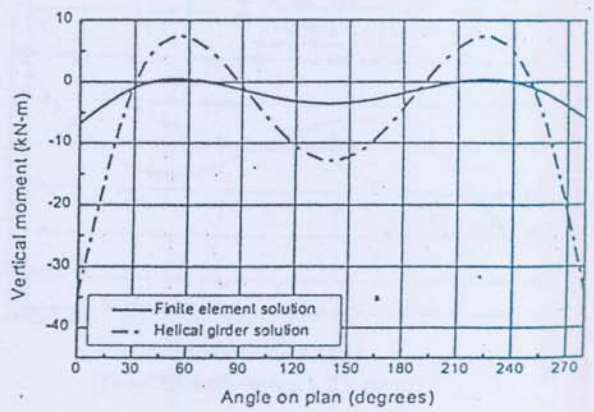

Figure 11: Vertical moment (Stair-II)

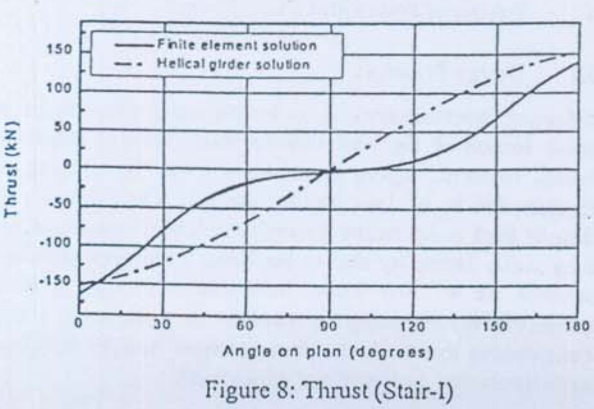

Figure 8: Thrust (Stair-I)

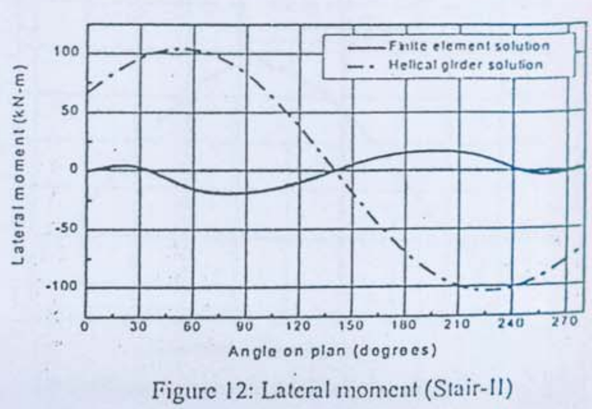

Figure 12: Lateral moment (Stair-II)

Figure 13: Torsion (Stair-II)

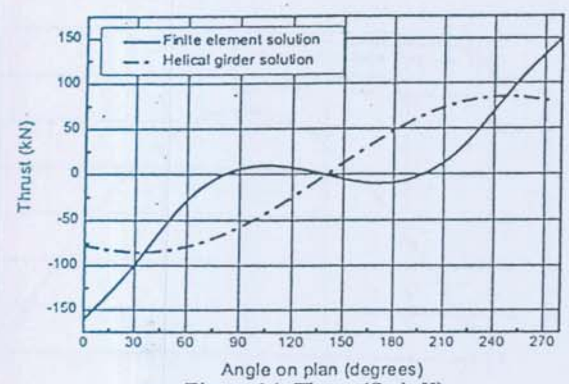

Figure 14: Thrust (Stair-II)

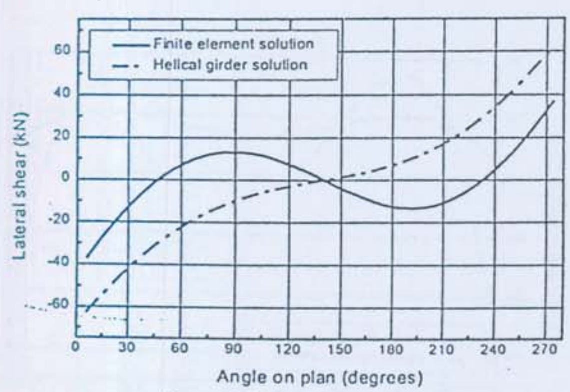

Figure 15: Lateral shear (Stair-II)

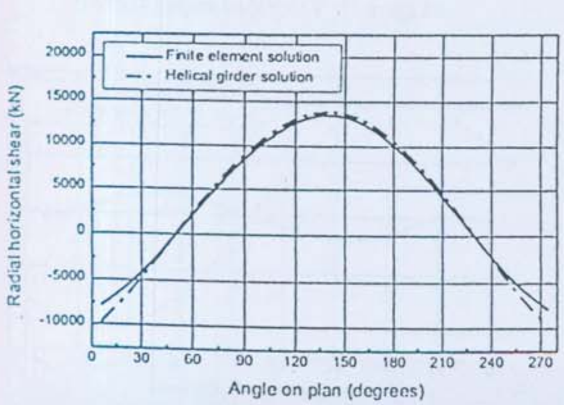

Figure 16: Radial horizontal shear (Stair-II)

5.2 A Comparative Assessment between FE Solution and Helical Girder Solution

From the comparative observations of the results of analyses of the prototype structures, the following significant points were noted:

1. The vertical moment is over estimated in the helical girder solution. In both Stair-I and Stair-II, the helical girder solution over estimated the vertical moment at the support by about six times (Figures 5 and 11). However, at the mid span, this over estimation was about three times for both the cases. The variation pattern of generated vertical moment diagram was found to be similar in both the approaches.

2. A large over estimation of lateral moment was observed in the helical girder solution. In case of Stair-I, the over estimation was to the extent of twelve times (Figure 6) and it was about six times in case of Stair-II (Figure 12). However, both approaches estimated the absence of any lateral moment at the mid span.

3. The torsion was found to be under estimated in the helical girder solution. In Stair-I, the maximum torsion was under estimated by a factor of more than two (Figure 7). In Stair-II, the under estimation of torsion was found to be less pronounced (Figure 13).

4. In Stair-I and Stair-II the maximum value of thrust was similar in FE approach (Figures 8 and 14). In case of Stair-II, the maximum thrust at the support was under estimated by a factor of nearly two with helical girder approach (Figure 14). However, in the both structures, the variation of thrust away from the support predicted by the helical girder solution indicated considerable over estimation.

5. From both the approaches, the lateral shear was found to be maximum at the support. But the helical girder solution was, once again, found to over estimate the forces. Whereas, in case of Stair-I, about three times over estimation was noticed (Figure 9), in the other case, this was about two times (Figure 15).

6. Both the approaches estimated almost the same value of radial horizontal shear (Figures 10 and 16).

6 Design of Helicoidal Stair Slabs

6.1 Design Principle

Adequate reinforcement is to be provided properly at every cross section of the stair slab to resist vertical moment and lateral moment. Again, concrete is weak in resisting direct tension due to its low tensile strength. On the other hand, despite high compressive strength, a slender concrete structure may suffer instability due to buckling accompanied with high stresses. So to resist thrust, helicoidal stair slab is designed conservatively assuming the steel to carry the entire tensile or compressive thrusts. Interaction between bi-axial bending and axial thrust was, however, not considered.

Section (degree*)	Amount of reinforcement (mm²) for							
	FE solution				Helical girder solution			
	Vertical moment	Thrust	Total requirement	Amount provided**	Vertical moment	Thrust	Total requirement	Amount provided**
0	-135	-903	1039	1161	-884	-839	1729	1729
18	-58	-645	703	1161	-265	-755	1013	1161
36	-32	-329	361	1161	-52	-613	671	1161
54	-39	-135	174	1161	-77	-439	510	1161
72	-52	-52	103	1161	-168	-226	394	1161
90	-58	-19	77	1161	-206	0.00	206	1161
108	-45	13	58	1161	-168	226	394	1161
126	-26	103	129	1161	-77	439	510	1161
144	-19	284	297	1161	-52	613	671	1161
162	-39	548	594	1161	-265	755	1013	1161
180	-90	768	858	1161	-884	839	1729	1729

* *Angular distance from bottom end,* ** *Considering temperature and shrinkage reinforcement requirement*

Table 2: Comparative estimation of reinforcement requirement for vertical moment and thrust.

It is not practicable to provide stirrups as shear reinforcement in helicoidial stair slabs that usually have a thin cross-section. Again, in such stair slabs, the provision of reinforcement for resisting excessive torque will not be very effective because of shallow section. This is why it is advisable to use appropriate waist thickness, for torsion and shear.

In order to prevent temperature and shrinkage cracking, the total amount of reinforcement to be provided along both the directions of slab (i.e. 'along the span' and 'across the span') must not be less than 0.20 percent of the gross concrete cross section [17].

6.2 Design Example

Based on the above stated design principle, the Stair-I was designed for factored dead and live loads (1.4DL + 1.7LL) in the ultimate strength design method following ACI code of practice [17] with steel and concrete having ultimate strength of 275 MPa and 17.5 MPa, respectively.

The designed reinforcement required to resist vertical moment and thrust at various sections along with the check for adequacy of temperature and shrinkage reinforcement (along the span) has been presented in Table 2. In addition to these reinforcements, the ACI recommended amount of reinforcement (0.20 percent of gross concrete cross section) has also to be provided across the width of the stair slab for control of temperature and shrinkage cracking. The reinforcement required to resist lateral moment at different sections has been summarised in Table 3.

The allowable shear force for the concrete section under the action of combined shear and torsion has been calculated to be 192 kN which is much higher than the developed ultimate lateral shear (29 kN). The torsion carrying capacity of this concrete section was calculated to be 243 kN-m against maximum ultimate torsion as 37 kN-m. So there is no need for providing any reinforcement for shear or torsion.

These figures clearly indicate a possibility of designing such structures for thinner waists with marginal savings. But it has to be kept in mind that helicoidal stair is a form of free standing stair, where a deflection criterion is important from serviceability point of view. Again, the stairs are situated in public buildings, where always remains a chance of over loading due to mass movement of pedestrian, which has not been covered in the present analysis.

Section (degree*)	Amount of reinforcement (mm²)	
	FE solution	Helical girder solution
0	-39	839
18	19	794
36	-39	677
54	-39	497
72	-19	258
90	0	0
108	19	-258
126	26	-497
144	19	-677
162	-32	-794
180	26	-839

* *The angular distance from bottom end*

Table 3: Comparative estimation of reinforcement requirement for lateral moment.

6.3 The Economy Attainable with FE Approach

The comparative illustrative design of Stair-I (Tables 2 and 3) indicates that the requirements for resisting vertical and lateral moments and thrust are significantly less in FE analysis than that in helical girder solution. However, in both approaches, the consideration of temperature and shrinkage reinforcement governs in all sections other than the supports, where about 33% saving of reinforcement can be achieved. From the design exercise, it also became evident that only around 7% of the traditionally used reinforcement is sufficient to take care of the lateral moment. Table 4 presents a comparative picture of component wise economy associated with FE approach. These figures reveal that about 47% savings on the reinforcement to be laid along the span can be achieved. However, considering the requirement of temperature and shrinkage reinforcement to be provided across the span, the overall economy of reinforcement stands around 17%.

7 Conclusions

The FE approach using thick shell element can analyse the helicoidal stair slabs without any geometric idealisation. It also takes the inherent structural efficiency of this shell structure into account. Thus a considerable economy of the reinforcement in resisting moments and thrust can be

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Method	Amount of reinforcement (kg) for			Total requirement (kg)
	Vertical moment and thrust	Lateral moment	Temp. & shrinkage reinforcement*	
FE	75.92	5.11	278.39	359.43
Helical girder	81.17	71.71	278.39	431.26

**Across the span*

Table 4: Comparative estimation of total reinforcement requirement in FE solution and helical girder solution.

achieved. The present case study indicated that around 47% economy of reinforcement for moment and thrust can be achieved with an overall economy of around 17%. These findings of the present investigations clearly demonstrate the potentials of the FE approach over the traditional helical girder solution for designing the helicoidal stair slabs in a cost-effective way. An extensive parametric study in terms of different geometric parameters has therefore been carried out to generalise the behaviour of this form of stair slab. Based on this sensitivity study, a new and economic design rationale for the helicoidal stair slab has been developed [20] with the powerful FE method of analysis by using curved thick shell element.

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